

Dust particles in mean motion resonances influenced by an interstellar gas flow

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ABSTRACT

The orbital evolution of a dust particle captured in a mean motion resonance with a planet in circular orbit under the action of the Poynting-Robertson effect, radial stellar wind and an interstellar gas flow of is investigated. The secular time derivative of Tisserand parameter is analytically derived for arbitrary orbit orientation. From the secular time derivative of Tisserand parameter a general relation between the secular time derivatives of eccentricity and inclination is obtained. In the planar case (the case when the initial dust particle position vector, initial dust particle velocity vector and interstellar gas velocity vector lie in the planet orbital plane) is possible to calculate directly the secular time derivative of eccentricity.

Using numerical integration of equation of motion we confirmed our analytical results in the three-dimensional case and also in the planar case. Evolutions of eccentricity of the dust particle captured in an exterior mean motion resonance under the action of the Poynting-Robertson effect, radial stellar wind for the cases with and without the interstellar gas flow are compared.

Qualitative properties of the orbital evolution in the planar case are determined. Two main groups of the secular orbital evolutions exist. In the first group the eccentricity and argument of perihelion approach to some values. In the second group the eccentricity oscillates and argument of perihelion rapidly shifts.

Key words: ISM: general – celestial mechanics – interplanetary medium

1 INTRODUCTION

For the analysis of dynamical evolution of dust particles in the vicinity of a star is necessary to take into account also non-gravitational effects. From the non-gravitational effects accelerations caused by the electromagnetic radiation and the corpuscular radiation (stellar wind) of the central star is most often considered. Influence of the electromagnetic radiation on the dynamics of dust grains is usually described using Poynting-Robertson (PR) effect (Poynting 1904; Robertson 1937; Klačka 1992, 2004; Klačka et al. 2012b). For the corpuscular radiation is usually assumed radial propagation from the central star (Whipple 1955). A more general form of the acceleration caused by the corpuscular radiation can be found in Klačka et al. (2012a). When orbital periods of the particle and a planet are in a ratio of small natural numbers mean motion resonances (MMRs) can occur. If the dust particle is captured in an MMR, changes of the semi-major axis caused by small non-gravitational effects are balanced by the resonant interaction with the planet's gravity field. Jackson & Zook (1989) predicted a ring of dust par-

ticles orbiting the Sun captured in mean motion resonances with planet Earth. This ring was confirmed by observations from satellites IRAS (Brownlee 1994; Dermott et al. 1994) and COBE (Reach et al. 1995). The paper Jackson & Zook (1989) was followed by many others who investigated behaviour of dust particles captured in MMRs during the last three decades (e.g. Weidenschilling & Jackson 1993; Marzari & Vanzani 1994; Šidlichovský & Nesvorný 1994; Liou & Zook 1995; Liou, Zook & Jackson 1995; Liou & Zook 1997, 1999; Moro-Martín & Malhotra 2002; Holmes et al. 2003; Kuchner & Holman 2003; Deller & Maddison 2005; Klačka, Koci-faj & Pástor 2005a,b; Krivov et al. 2007; Stark & Kuchner 2008; Reach 2010; Mustill & Wyatt 2011; Ertel, Wolf & Rodmann 2012). If the planet moves in a circular orbit around the Sun and mass of the particle is negligible in comparison with mass of the Sun and also in comparison with mass of the planet, we have a special gravitational problem of three bodies. This gravitational problem is called the circular restricted three-body problem (CR3BP) in celestial mechanics. An analytic expression for the secular time derivative of orbital eccentricity of the dust particle captured in an MMR in the planar CR3BP with the PR effect and the radial solar wind was found in Liou & Zook (1997). Due to the rotation

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of the star stellar wind can be non-radial in general. According to Helios 2 measurements (Bruno et al. 2003) the angle between the radial direction and the direction of solar wind velocity is approximately constant (at least for the distances covered by observations). The secular time derivative of orbital eccentricity of the dust particle captured in an MMR in the planar CR3BP with the PR effect and such non-radial solar wind was calculated in Klačka et al. (2008) and all possibilities of the secular eccentricity evolution were analytically determined in Pástor et al. (2009b). The secular evolution of orbital eccentricity and argument of perihelion of the dust particle captured in an MMR in the planar circular and elliptical restricted three-body problem with the PR effect was numerically investigated in Pástor, Klačka & Kómar (2009a).

Interstellar medium atoms penetrate into the Solar System due to relative motion of the Solar System with respect to the interstellar medium. These approaching atoms form an interstellar gas flow (IGF). Influence of this IGF on motion of dust particles orbiting a star was mentioned already in Whipple (1955). Motion of dust particles orbiting the Sun influenced by the IGF was analytically and numerically investigated in Scherer (2000). When the IGF velocity vector lies in the orbital plane of the particle and the particle is under the action of the PR effect, radial solar wind, and an IGF, then the motion occurs in a plane. Scherer (2000) has correctly described qualitative properties of the shift of perihelion in the planar case despite of the fact that his calculations contain several errors. Secular time derivatives of semimajor axis, eccentricity and argument of perihelion in the planar case were calculated in Klačka et al. (2009). In Pástor, Klačka & Kómar (2011) secular time derivatives of all Keplerian orbital elements for arbitrary orientation of the orbit with respect to interstellar gas velocity vector were calculated. The secular time derivatives of orbital elements were in Pástor et al. (2011) derived under the assumptions (a) that the acceleration caused by the IGF is small compared to the gravitation of a central object, (b) that the speed of the IGF is large in comparison with the speed of the dust particle (speeds are determined with respect to the central object) and (c) that the speed of the IGF is large also in comparison with the mean thermal speed of the gas in the flow (Mach number $\gg 1$). Under these assumptions the IGF always causes decrease of the secular semimajor axis of the dust particle. This decrease of the secular semimajor axis was confirmed analytically in Belyaev & Rafikov (2010) and using numerical integrations in Pástor et al. (2011), Marzari & Thébault (2011) and Marzari (2012). The acceleration of the dust particle caused by the IGF depends on the drag coefficient which is, for the given particle and the flow of interstellar gas, a specific function of the relative speed of the dust particle with respect to the interstellar gas (Baines, Williams & Asebiomo 1965). The drag coefficient was in Scherer (2000), Klačka et al. (2009), Pástor et al. (2011), Marzari & Thébault (2011) and Marzari (2012) taken into account as a constant. This is a consequence of assumption (c). The derivation of the secular time derivatives of all Kep-

lerian orbital elements was generalised by adding variability of the drag coefficient during orbit in Pástor (2012b) using a method applied in Belyaev & Rafikov (2010) for the secular time derivative of semimajor axis. In view of assumptions this means that the secular time derivatives were derived under the assumptions (a), (b) and assumption (c) was replaced by assumption that the mean thermal speed of the gas in the flow is not close to zero (see Baines et al. 1965). Also under these assumptions the secular semimajor axis of particle's orbit always decreases under the action of the IGF. The minimal and maximal values of the decrease of the semimajor axis were also determined in Pástor (2012b).

In this work we use the secular time derivatives of orbital elements caused by an IGF derived in Pástor (2012b) to obtain some basic properties of orbital evolution of the dust particle captured in an MMR with a planet in circular orbit.

2 SECULAR ORBITAL EVOLUTION IN MMRS

Within the framework of the CR3BP under the assumptions that the particle is far enough from the planet and mass of the planet is negligible with respect to the mass of the Sun F . F. Tisserand found a quantity which remains constant during the motion of the particle (Tisserand 1896, e.g. Brouwer & Clemence 1961)

$$C_{T0} = \frac{1}{2a} + \sqrt{\frac{a(1-e^2)}{a_P^3}} \cos i, \quad (1)$$

where a is the semimajor axis of the particle orbit, e the eccentricity of the particle orbit, i is the inclination of the particle orbit with respect to the planet orbital plane and a_P is the semimajor axis of the planet orbit. If we take into account also non-gravitational effects, Tisserand parameter will no longer be constant, in general. From the non-gravitational effects the electromagnetic radiation of the Sun in the form of the PR effect is most often considered. If we add Keplerian term of the PR effect to the central Keplerian acceleration of the Sun, Tisserand parameter obtains the following form (cf. Liou & Zook 1997)

$$C_T = \frac{1-\beta}{2a_\beta} + \sqrt{\frac{(1-\beta)a_\beta(1-e_\beta^2)}{a_P^3}} \cos i_\beta. \quad (2)$$

The parameter β is defined as the ratio of the electromagnetic radiation pressure force and the gravitational force between the Sun and the particle at rest with respect to the Sun

$$\beta = \frac{3L_\odot \bar{Q}'_{pr}}{16\pi c \mu R_\odot}. \quad (3)$$

Here, L_\odot is the solar luminosity, \bar{Q}'_{pr} is the dimensionless efficiency factor for radiation pressure averaged over the solar spectrum and calculated for the radial direction ($\bar{Q}'_{pr} = 1$ for a perfectly absorbing sphere), c is the speed of light in vacuum, $\mu = GM_\odot$, G is the gravitational constant, M_\odot

is the mass of the Sun, and R is radius of the dust particle with the mass density ρ . Subscript β in Eq. (2) denotes that osculating orbital elements are calculated using acceleration $-\mu(1-\beta)\vec{r}/r^3$ as the central Keplerian acceleration. Here, \vec{r} is the position vector of the dust particle with respect to the Sun and $r = |\vec{r}|$.

The particle is in an MMR with the planet when the ratio of their mean motions is equal to the ratio of two small natural numbers. For an exterior q -th order MMR we have $n_P/n = (p+q)/p$ and $n_P/n = p/(p+q)$ for an interior q -th order MMR. Here, p and q are two small natural numbers, n_P is the mean motion of the planet and n is the mean motion of the particle. The special case mean motion 1/1 resonance corresponds to $q = 0$. If the dust particle is captured in the MMR, then the semimajor axis of the particle's orbit librates around a constant value. This can be written in the following form

$$\left\langle \frac{da_\beta}{dt} \right\rangle = 0. \quad (4)$$

Averaging in Eq. (4) is over a period of the resonant libration. Using Kepler's Third Law we can obtain a relation between the semimajor axis of the planet and the semimajor axis of the particle captured in the MMR. We have

$$G(M_\odot + M_P) = n_P^2 a_P^3, \quad (5)$$

$$GM_\odot(1-\beta) = n^2 a_\beta^3, \quad (6)$$

where M_P is the mass of the planet. Putting Eq. (5) into Eq. (6) (with assumption $M_P \ll M_\odot$) we find the relation

$$a_\beta = a_P(1-\beta)^{1/3} \left(\frac{n_P}{n} \right)^{2/3}. \quad (7)$$

Now, we admit that the dust particle is under the action of arbitrary non-gravitational effects for which secular time derivatives of the Keplerian orbital elements can be determined. Because, the secular semimajor axis is constant, the gravitational influence of the planet (subscript G) must compensate changes of the semimajor axis caused by the non-gravitational effects (subscript EF)

$$\left\langle \frac{da_\beta}{dt} \right\rangle = \left\langle \frac{da_\beta}{dt} \right\rangle_G + \left\langle \frac{da_\beta}{dt} \right\rangle_{EF} = 0. \quad (8)$$

We have also

$$\left\langle \frac{de_\beta}{dt} \right\rangle = \left\langle \frac{de_\beta}{dt} \right\rangle_G + \left\langle \frac{de_\beta}{dt} \right\rangle_{EF}, \quad (9)$$

$$\left\langle \frac{di_\beta}{dt} \right\rangle = \left\langle \frac{di_\beta}{dt} \right\rangle_G + \left\langle \frac{di_\beta}{dt} \right\rangle_{EF}. \quad (10)$$

For a particle with $\beta = 0$ (e.g., an asteroid) in the planar CR3BP ($i_\beta = 0$ and $di_\beta/dt = 0$) captured in an MMR the eccentricity oscillates around a constant value, because the Tisserand parameter (Eq. 1) also oscillates around a constant value. Now we find a relation between the secular evolution of eccentricity and inclination for the dust particle under the action of the non-gravitational effects captured in an MMR. The total differential of the Tisserand parameter

is

$$\begin{aligned} \left\langle \frac{dC_T}{dt} \right\rangle &= \frac{\partial C_T}{\partial a_\beta} \left\langle \frac{da_\beta}{dt} \right\rangle + \frac{\partial C_T}{\partial e_\beta} \left\langle \frac{de_\beta}{dt} \right\rangle + \frac{\partial C_T}{\partial i_\beta} \left\langle \frac{di_\beta}{dt} \right\rangle \\ &= \frac{\partial C_T}{\partial a_\beta} \left(\left\langle \frac{da_\beta}{dt} \right\rangle_G + \left\langle \frac{da_\beta}{dt} \right\rangle_{EF} \right) \\ &\quad + \frac{\partial C_T}{\partial e_\beta} \left(\left\langle \frac{de_\beta}{dt} \right\rangle_G + \left\langle \frac{de_\beta}{dt} \right\rangle_{EF} \right) \\ &\quad + \frac{\partial C_T}{\partial i_\beta} \left(\left\langle \frac{di_\beta}{dt} \right\rangle_G + \left\langle \frac{di_\beta}{dt} \right\rangle_{EF} \right). \end{aligned} \quad (11)$$

When the particle is far enough from the planet and the mass of the planet is negligible with respect to the mass of the Sun, then the planet gravitation does not change value of the Tisserand parameter. Hence

$$\begin{aligned} \left\langle \frac{dC_T}{dt} \right\rangle &= \frac{\partial C_T}{\partial a_\beta} \left\langle \frac{da_\beta}{dt} \right\rangle_{EF} + \frac{\partial C_T}{\partial e_\beta} \left\langle \frac{de_\beta}{dt} \right\rangle_{EF} \\ &\quad + \frac{\partial C_T}{\partial i_\beta} \left\langle \frac{di_\beta}{dt} \right\rangle_{EF}. \end{aligned} \quad (12)$$

Using the condition that the particle is in an MMR (Eq. 4) and Eq. (12) in Eq. (11) we obtain

$$\begin{aligned} \left\langle \frac{dC_T}{dt} \right\rangle &= \frac{\partial C_T}{\partial e_\beta} \left\langle \frac{de_\beta}{dt} \right\rangle + \frac{\partial C_T}{\partial i_\beta} \left\langle \frac{di_\beta}{dt} \right\rangle \\ &= \frac{\partial C_T}{\partial a_\beta} \left\langle \frac{da_\beta}{dt} \right\rangle_{EF} + \frac{\partial C_T}{\partial e_\beta} \left\langle \frac{de_\beta}{dt} \right\rangle_{EF} \\ &\quad + \frac{\partial C_T}{\partial i_\beta} \left\langle \frac{di_\beta}{dt} \right\rangle_{EF}. \end{aligned} \quad (13)$$

Eq. (13) represents the relation which must hold between the secular time derivatives of eccentricity and inclination of the dust particle captured in the MMR under the action of the non-gravitational effects. For the secular time derivative of eccentricity we obtain

$$\begin{aligned} \left\langle \frac{de_\beta}{dt} \right\rangle &= \frac{\partial C_T / \partial a_\beta}{\partial C_T / \partial e_\beta} \left\langle \frac{da_\beta}{dt} \right\rangle_{EF} + \left\langle \frac{de_\beta}{dt} \right\rangle_{EF} \\ &\quad - \frac{\partial C_T / \partial i_\beta}{\partial C_T / \partial e_\beta} \left\langle \frac{di_\beta}{dt} \right\rangle_G. \end{aligned} \quad (14)$$

Calculation of the partial derivatives yields

$$\begin{aligned} \left\langle \frac{de_\beta}{dt} \right\rangle &= \frac{1-e_\beta^2}{2a_\beta e_\beta} \left[\frac{(1-\beta)^{1/2}}{(1-e_\beta^2)^{1/2} \cos i_\beta} \left(\frac{a_P}{a_\beta} \right)^{3/2} - 1 \right] \\ &\quad \times \left\langle \frac{da_\beta}{dt} \right\rangle_{EF} + \left\langle \frac{de_\beta}{dt} \right\rangle_{EF} \\ &\quad - \frac{1-e_\beta^2}{e_\beta} \tan i_\beta \left\langle \frac{di_\beta}{dt} \right\rangle_G. \end{aligned} \quad (15)$$

Due to the derivation formulated for arbitrary non-gravitational effects with known secular time derivatives of orbital elements Eqs. (13) and (15) are generalisations of the results obtained by Liou & Zook (1997).

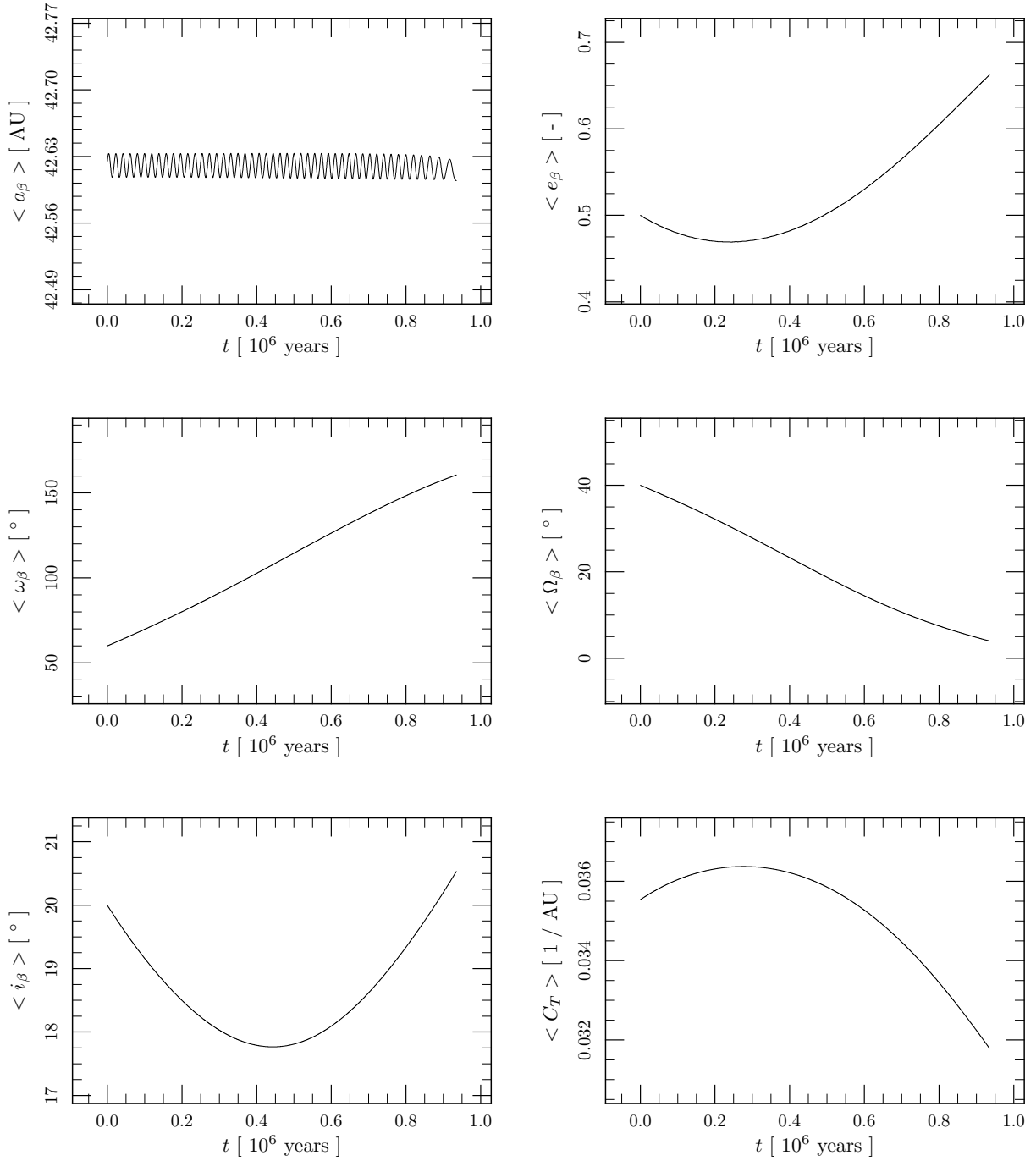


Figure 1. Evolutions of the orbit averaged semi-major axis, eccentricity, argument of perihelion, longitude of the ascending node, inclination and Tisserand parameter of a dust particle with $R = 2 \mu\text{m}$, $\varrho = 1 \text{ g/cm}^3$, and $\tilde{Q}'_{pr} = 1$ captured in an exterior mean motion orbital 2/1 resonance with Neptune under the action of the PR effect, radial solar wind and IGF.

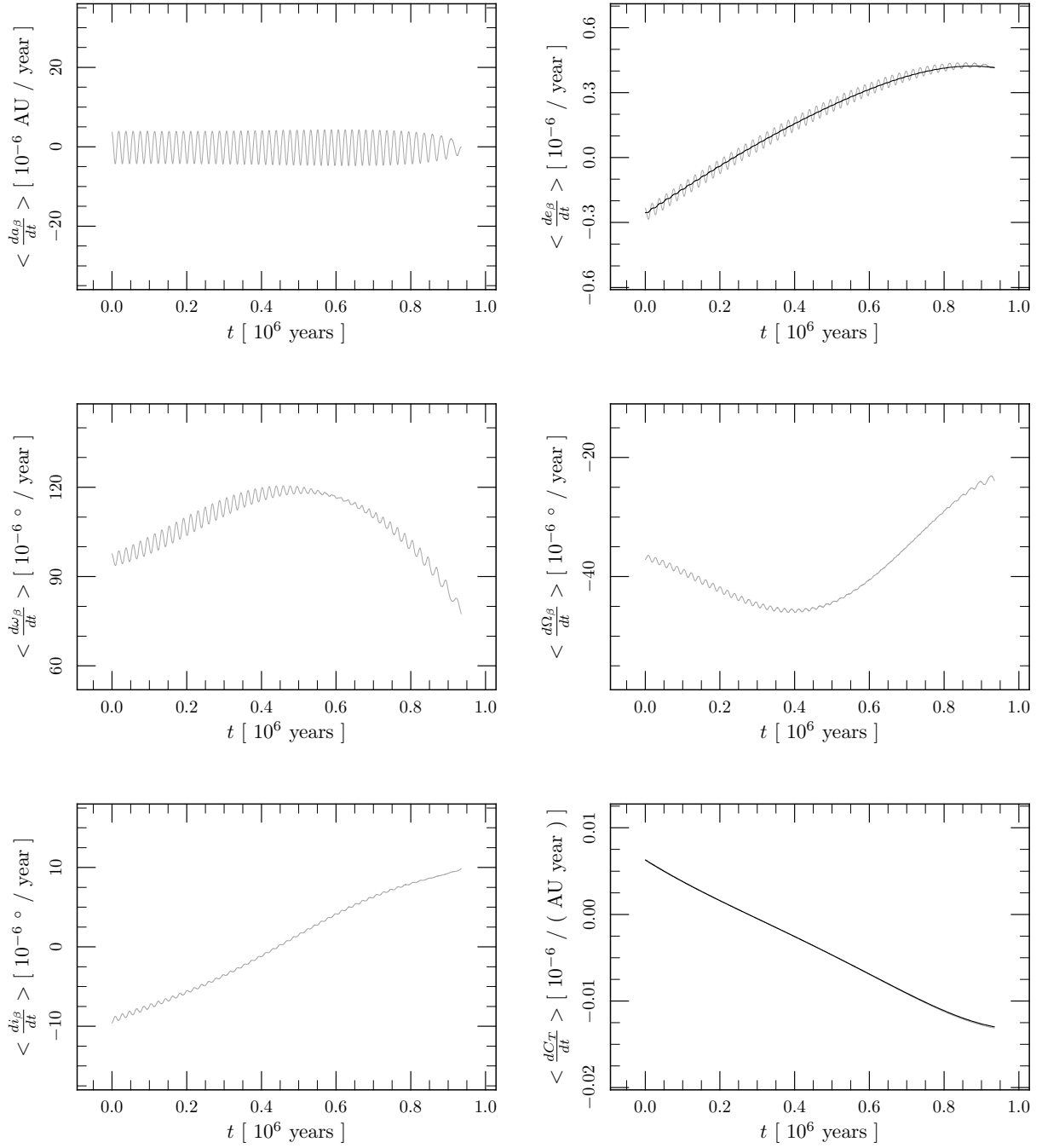


Figure 2. Evolutions of the orbit averaged time derivative of semi-major axis, eccentricity, argument of perihelion, longitude of the ascending node, inclination and Tisserand parameter during the numerical solution of Eq. (30) depicted in Fig. 1 (grey line). The evolution of the orbit averaged time derivative of Tisserand parameter is compared with the secular time derivative of Tisserand parameter obtained from Eq. (28) (black line). The evolution of the orbit averaged time derivative of eccentricity is compared with the secular time derivative of eccentricity calculated from Eq. (28) using the numerical values of $\langle di_\beta/dt \rangle$ depicted in the bottom left panel (black line).

3 INFLUENCE OF AN IGF ON ORBITAL EVOLUTION OF DUST PARTICLES IN MMRS

In this section we use the analytical theory from the previous section in order to find equations for the secular time derivatives of the particle's orbital elements in the case when the particle, captured in an MMR, is under the action of the PR effect, radial solar wind and IGF. The secular changes caused by the IGF will be described using the secular time derivatives of orbital elements derived in Pástor (2012b). For the PR effect and radial solar wind the standard expressions will be used (Wyatt & Whipple 1950; Klačka et al. 2012a). The secular time derivatives of the orbital elements caused by these effects then are

$$\begin{aligned} \left\langle \frac{da_\beta}{dt} \right\rangle_{EF} &= -\beta \frac{\mu}{c} \left(1 + \frac{\eta}{\bar{Q}'_{pr}} \right) \frac{2 + 3e_\beta^2}{a_\beta(1 - e_\beta^2)^{3/2}} \\ &\quad - \sum_i 2a_\beta c_{0i} \gamma_i v_F^2 \sqrt{\frac{p_\beta}{\mu(1 - \beta)}} \sigma_\beta \\ &\quad \times \left[1 + \frac{g_i}{v_F^2} \frac{1 - \sqrt{1 - e_\beta^2}}{e_\beta^2} \right] \\ &\quad \times \left(S_\beta^2 + I_\beta^2 \sqrt{1 - e_\beta^2} \right), \end{aligned} \quad (16)$$

$$\begin{aligned} \left\langle \frac{de_\beta}{dt} \right\rangle_{EF} &= -\beta \frac{\mu}{c} \left(1 + \frac{\eta}{\bar{Q}'_{pr}} \right) \frac{5e_\beta}{2a_\beta^2(1 - e_\beta^2)^{1/2}} \\ &\quad + \sum_i c_{0i} \gamma_i v_F \sqrt{\frac{p_\beta}{\mu(1 - \beta)}} \\ &\quad \times \left[\frac{3I_\beta}{2} + \frac{\sigma_\beta g_i (I_\beta^2 - S_\beta^2)(1 - e_\beta^2)}{v_F e_\beta^3} \right] \\ &\quad \times \left(1 - \frac{e_\beta^2}{2} - \sqrt{1 - e_\beta^2} \right), \end{aligned} \quad (17)$$

$$\begin{aligned} \left\langle \frac{d\omega_\beta}{dt} \right\rangle_{EF} &= \sum_i \frac{c_{0i} \gamma_i v_F}{2} \sqrt{\frac{p_\beta}{\mu(1 - \beta)}} \left\{ -\frac{3S_\beta}{e_\beta} \right. \\ &\quad + \frac{\sigma_\beta g_i S_\beta I_\beta}{v_F e_\beta^4} \left[e_\beta^4 - 6e_\beta^2 + 4 - 4(1 - e_\beta^2)^{3/2} \right] \\ &\quad + C_\beta \frac{\cos i_\beta}{\sin i_\beta} \left[\frac{3e_\beta \sin \omega_\beta}{1 - e_\beta^2} - \frac{\sigma_\beta g_i}{v_F} \right. \\ &\quad \times \left. \left. \left(S_\beta \cos \omega_\beta - I_\beta \sin \omega_\beta \right) \right] \right\}, \end{aligned} \quad (18)$$

$$\begin{aligned} \left\langle \frac{d\Omega_\beta}{dt} \right\rangle_{EF} &= \sum_i \frac{c_{0i} \gamma_i v_F C_\beta}{2 \sin i_\beta} \sqrt{\frac{p_\beta}{\mu(1 - \beta)}} \\ &\quad \times \left[-\frac{3e_\beta \sin \omega_\beta}{1 - e_\beta^2} + \frac{\sigma_\beta g_i}{v_F} \right. \end{aligned}$$

$$\left. \times \left(S_\beta \cos \omega_\beta - I_\beta \sin \omega_\beta \right) \right], \quad (19)$$

$$\begin{aligned} \left\langle \frac{di_\beta}{dt} \right\rangle_{EF} &= -\sum_i \frac{c_{0i} \gamma_i v_F C_\beta}{2} \sqrt{\frac{p_\beta}{\mu(1 - \beta)}} \\ &\quad \times \left[\frac{3e_\beta \cos \omega_\beta}{1 - e_\beta^2} + \frac{\sigma_\beta g_i}{v_F} \right. \\ &\quad \times \left. \left. \left(S_\beta \sin \omega_\beta + I_\beta \cos \omega_\beta \right) \right] \right], \end{aligned} \quad (20)$$

where ω_β is the argument of perihelion and Ω_β is the longitude of the ascending node. The sums in Eqs. (16)-(20) run over all particle species i in the IGF. η is the ratio of solar wind energy to electromagnetic solar energy, both radiated per unit of time

$$\eta = \frac{4\pi r^2 u}{L_\odot} \sum_j n_{sw\ j} m_{sw\ j} c^2, \quad (21)$$

here, u is the speed of the solar wind with respect to the Sun, $u = 450$ km/s, $m_{sw\ j}$ and $n_{sw\ j}$, are the masses and concentrations of the solar wind particles of j -th type at a distance r from the Sun, respectively. $\eta = 0.38$ for the Sun (Klačka et al. 2012a). c_{0i} is the drag coefficient for the dust particle at rest with respect to the Sun (Baines et al. 1965)

$$\begin{aligned} c_{0i} &= \frac{1}{\sqrt{\pi}} \left(\frac{1}{s_{0i}} + \frac{1}{2s_{0i}^3} \right) e^{-s_{0i}^2} \\ &\quad + \left(1 + \frac{1}{s_{0i}^2} - \frac{1}{4s_{0i}^4} \right) \text{erf}(s_{0i}) \\ &\quad + (1 - \delta_i) \left(\frac{T_d}{T_i} \right)^{1/2} \frac{\sqrt{\pi}}{3s_{0i}}, \end{aligned} \quad (22)$$

here, $\text{erf}(s_i)$ is the error function $\text{erf}(s_{0i}) = 2/\sqrt{\pi} \int_0^{s_{0i}} e^{-t^2} dt$, δ_i is the fraction of impinging particles specularly reflected at the surface (for the resting particles, there is assumed diffuse reflection) (Baines et al. 1965; Gustafson 1994), T_d is the temperature of the dust grain, and T_i is the temperature of the i -th gas component, s_{0i} is the molecular speed ratio for the dust particle at rest with respect to the Sun (Mach number)

$$s_{0i} = \sqrt{\frac{m_i}{2kT_i}} v_F, \quad (23)$$

m_i is the mass of the neutral atoms of type i , k is the Boltzmann's constant, $\vec{v}_F = (v_{FX}, v_{FY}, v_{FZ})$ is the velocity of the IGF in the frame associated with the Sun, $v_F = |\vec{v}_F|$, γ_i is the collision parameter,

$$\gamma_i = n_i \frac{m_i}{m} A, \quad (24)$$

here, n_i is the concentration of the interstellar neutral atoms of type i , and $A = \pi R^2$ is the geometrical cross section of the dust particle with mass m , $p_\beta = a_\beta(1 - e_\beta^2)$,

$$\sigma_\beta = \frac{\sqrt{\mu(1-\beta)/p_\beta}}{v_F}, \quad (25)$$

the parameter g_i describes variability of the drag coefficient (Pástor 2012b)

$$g_i = \frac{1}{c_{0i}} \left[\frac{1}{\sqrt{\pi}} \left(\frac{1}{s_{0i}} - \frac{3}{2s_{0i}^3} \right) e^{-s_{0i}^2} + \left(1 - \frac{1}{s_{0i}^2} + \frac{3}{4s_{0i}^4} \right) \text{erf}(s_{0i}) \right] \quad (26)$$

and

$$\begin{aligned} S_\beta &= (\cos \Omega_\beta \cos \omega_\beta - \sin \Omega_\beta \sin \omega_\beta \cos i_\beta) v_{FX} \\ &\quad + (\sin \Omega_\beta \cos \omega_\beta + \cos \Omega_\beta \sin \omega_\beta \cos i_\beta) v_{FY} \\ &\quad + \sin \omega_\beta \sin i_\beta v_{FZ}, \\ I_\beta &= (-\cos \Omega_\beta \sin \omega_\beta - \sin \Omega_\beta \cos \omega_\beta \cos i_\beta) v_{FX} \\ &\quad + (-\sin \Omega_\beta \sin \omega_\beta + \cos \Omega_\beta \cos \omega_\beta \cos i_\beta) v_{FY} \\ &\quad + \cos \omega_\beta \sin i_\beta v_{FZ}, \\ C_\beta &= \sin \Omega_\beta \sin i_\beta v_{FX} - \cos \Omega_\beta \sin i_\beta v_{FY} \\ &\quad + \cos i_\beta v_{FZ}. \end{aligned} \quad (27)$$

If we use Eqs. (16), (17) and (20) in Eq. (13), we obtain

$$\begin{aligned} \left\langle \frac{dC_T}{dt} \right\rangle &= \frac{\partial C_T}{\partial e_\beta} \left\langle \frac{de_\beta}{dt} \right\rangle + \frac{\partial C_T}{\partial i_\beta} \left\langle \frac{di_\beta}{dt} \right\rangle \\ &= \beta \frac{\mu}{c} \left(1 + \frac{\eta}{Q'_{pr}} \right) \sqrt{\frac{1-\beta}{a_P^3 a^3}} \\ &\quad \times \left[\frac{(1-\beta)^{1/2} (2 + 3e_\beta^2)}{2(1-e_\beta^2)^{3/2}} \left(\frac{a_P}{a} \right)^{3/2} - \cos i_\beta \right] \\ &\quad + \sum_i c_{0i} \gamma_i v_F^2 \sqrt{\frac{p_\beta}{\mu(1-\beta)}} \\ &\quad \times \left\{ \sqrt{\frac{(1-\beta)p_\beta}{a_P^3}} \left[\frac{3}{2} \frac{1}{v_F} \frac{e_\beta}{1-e_\beta^2} \right. \right. \\ &\quad \times (C_\beta \cos \omega_\beta \sin i_\beta - I_\beta \cos i_\beta) \\ &\quad - \sigma_\beta \cos i_\beta + \frac{1}{2} \frac{\sigma_\beta g_i}{v_F^2} \left(C_\beta (S_\beta \sin \omega_\beta \right. \\ &\quad \left. \left. + I_\beta \cos \omega_\beta) \sin i_\beta - (S_\beta^2 + I_\beta^2) \cos i_\beta \right) \right] \\ &\quad \left. + \frac{1-\beta}{a_\beta} \sigma_\beta \left[1 + \frac{g_i}{v_F^2} \frac{1-\sqrt{1-e_\beta^2}}{e_\beta^2} \right. \right. \\ &\quad \left. \left. \times (S_\beta^2 + I_\beta^2 \sqrt{1-e_\beta^2}) \right] \right\}. \end{aligned} \quad (28)$$

This is the relation between $\langle de_\beta/dt \rangle$ and $\langle di_\beta/dt \rangle$ which must hold during the orbital evolution of the dust particle captured in an MMR. When the position vector and the velocity vector of the dust particle lie in the planet orbital plane and the interstellar gas velocity vector lies also in this

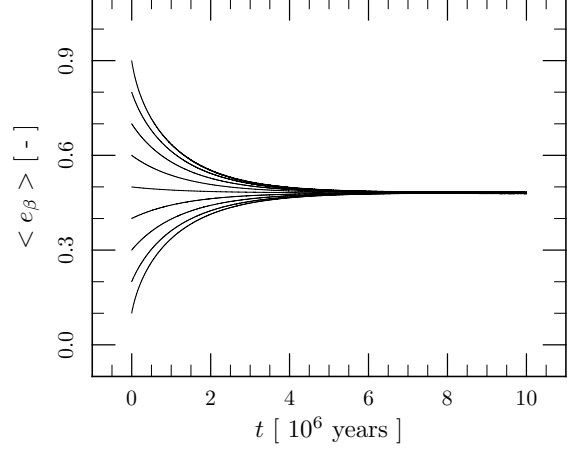


Figure 3. Evolutions of eccentricity of a dust particle with $R = 2 \mu\text{m}$, $\rho = 1 \text{ g/cm}^3$, and $\bar{Q}'_{pr} = 1$ captured in an exterior mean motion orbital 2/1 resonance with Neptune under the action of the PR effect and radial solar wind for various initial eccentricities.

plane, the dust particle's motion is coplanar. In this case we have $i_\beta = 0$ and $\langle di_\beta/dt \rangle = 0$. Using these assumptions we obtain for the secular time derivative of eccentricity from Eq. (28) and Eq. (7)

$$\begin{aligned} \left\langle \frac{de_\beta}{dt} \right\rangle &= \beta \frac{\mu}{c} \left(1 + \frac{\eta}{Q'_{pr}} \right) \frac{(1-e_\beta^2)^{1/2}}{a_\beta^2 e_\beta} \\ &\quad \times \left[1 - \frac{2 + 3e_\beta^2}{2(1-e_\beta^2)^{3/2}} \frac{n}{n_P} \right] \\ &\quad + \sum_i c_{0i} \gamma_i v_F^2 \sqrt{\frac{p_\beta}{\mu(1-\beta)}} \\ &\quad \times \left\{ \frac{3}{2} \frac{I_\beta}{v_F} + \frac{1-e_\beta^2}{e_\beta} \sigma_\beta \left[1 + \frac{1}{2} \frac{g_i}{v_F^2} (S_\beta^2 + I_\beta^2) \right] \right. \\ &\quad - \frac{(1-e_\beta^2)^{1/2}}{e_\beta} \frac{n}{n_P} \sigma_\beta \left[1 + \frac{g_i}{v_F^2} \frac{1-\sqrt{1-e_\beta^2}}{e_\beta^2} \right. \\ &\quad \left. \left. \times (S_\beta^2 + I_\beta^2 \sqrt{1-e_\beta^2}) \right] \right\}. \end{aligned} \quad (29)$$

4 NUMERICAL RESULTS

In this section we want to use numerical solutions of equation of motion in order to compare numerical results with the analytical results derived in the previous section and to find some main properties of dust particle orbital evolution in an MMR with a planet under the action of the solar radiation and IGF.

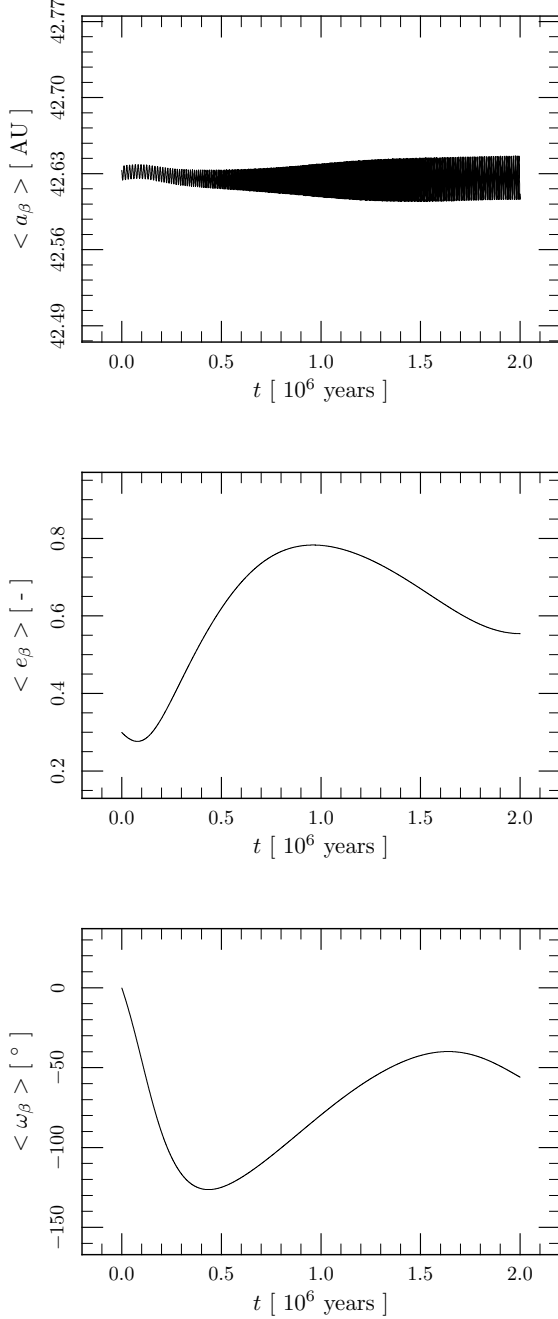


Figure 4. Evolutions of the orbit averaged semi-major axis, eccentricity and argument of perihelion of a dust particle with $R = 2 \mu\text{m}$, $\varrho = 1 \text{ g/cm}^3$, and $\bar{Q}'_{pr} = 1$ captured in an exterior mean motion orbital 2/1 resonance with Neptune under the action of the PR effect, radial solar wind and IGF in the planar case.

4.1 Equation of motion

If we consider the gravitation of the Sun, gravitation of one planet, PR effect, radial solar wind and IGF, then equation of motion of the dust particle has form (Klačka et al. 2012a,b; Baines et al. 1965)

$$\begin{aligned} \frac{d\vec{v}}{dt} = & -\frac{\mu}{r^2} (1 - \beta) \vec{e}_R \\ & - \beta \frac{\mu}{r^2} \left(1 + \frac{\eta}{\bar{Q}'_{pr}} \right) \left(\frac{\vec{v} \cdot \vec{e}_R}{c} \vec{e}_R + \frac{\vec{v}}{c} \right) \\ & - \sum_i c_{Di} \gamma_i |\vec{v} - \vec{v}_F| (\vec{v} - \vec{v}_F) \\ & - \frac{GM_P}{|\vec{r} - \vec{r}_P|^3} (\vec{r} - \vec{r}_P) \\ & - \frac{GM_P}{|\vec{r}_P|^3} \vec{r}_P, \end{aligned} \quad (30)$$

where $\vec{v} = d\vec{r}/dt$ is the velocity of the dust particle, $\vec{e}_R = \vec{r}/r$ is the radial unit vector, c_{Di} is the drag coefficient (Baines et al. 1965)

$$\begin{aligned} c_{Di} = & \frac{1}{\sqrt{\pi}} \left(\frac{1}{s_i} + \frac{1}{2s_i^3} \right) e^{-s_i^2} \\ & + \left(1 + \frac{1}{s_i^2} - \frac{1}{4s_i^4} \right) \text{erf}(s_i) \\ & + (1 - \delta_i) \left(\frac{T_d}{T_i} \right)^{1/2} \frac{\sqrt{\pi}}{3s_i}, \end{aligned} \quad (31)$$

s_i is the molecular speed ratio

$$s_i = \sqrt{\frac{m_i}{2kT_i}} |\vec{v} - \vec{v}_F| \quad (32)$$

and \vec{r}_P is the position vector of the planet with respect to the Sun.

4.2 Comparing analytical and numerical results in the three-dimensional case

From numerical solution of Eq. (30) the orbit averaged time derivative of Tisserand parameter can be obtained. We used the numerical solution of Eq. (30) and Eq. (28) for comparison of the numerical and analytical values of the secular time derivatives of Tisserand parameter. In the IGF we considered the primary and secondary populations of neutral hydrogen atoms and neutral helium atoms. The primary population of neutral hydrogen atoms and neutral helium atoms represent the original atoms of the IGF which penetrate into the heliosphere. The secondary population of neutral hydrogen atoms comprises the former protons from the IGF that acquired electrons from interstellar H° between the bow shock and the heliopause (Frisch et al. 2009; Alouani-Bibi et al. 2011). We adopted the following parameters for these components in the IGF. $n_1 = 0.059 \text{ cm}^{-3}$ and $T_1 = 6100 \text{ K}$ for the primary population of neutral hydrogen (Frisch et al. 2009), $n_2 = 0.059 \text{ cm}^{-3}$ and $T_2 = 16500 \text{ K}$ for the secondary population of neutral hydrogen (Frisch et al. 2009)

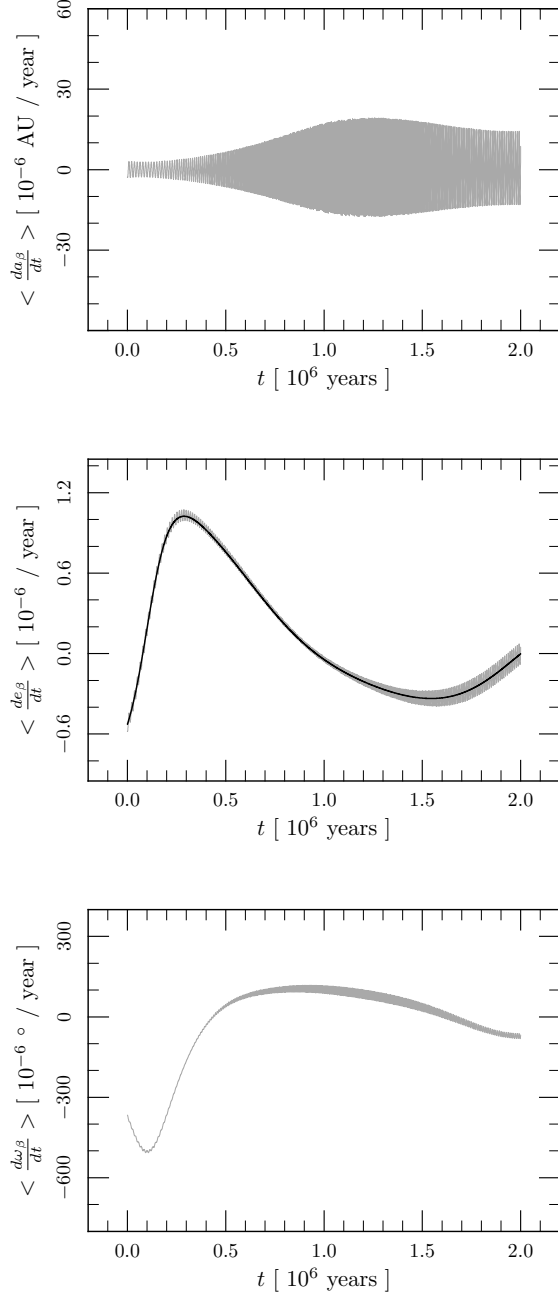


Figure 5. Evolutions of the orbit averaged time derivatives of semi-major axis, eccentricity and argument of perihelion during the numerical solution of Eq. (30) depicted in Fig. 4 (grey line). The evolution of the orbit averaged time derivative of eccentricity is compared with the secular time derivative of eccentricity obtained from Eq. (29) (black line).

and finally $n_3 = 0.015 \text{ cm}^{-3}$ and $T_3 = 6300 \text{ K}$ for the neutral helium (Lallement et al. 2005). We have assumed that the interstellar gas velocity vector is equal for all components and identical to the velocity vector of the neutral helium entering the Solar system. The neutral helium enter the Solar system with a speed of about $v_F = 26.3 \text{ km s}^{-1}$ (Lallement et al. 2005), and arrive from the direction of $\lambda_{ecl} = 254.7^\circ$ (heliocentric ecliptic longitude) and $\beta_{ecl} = 5.2^\circ$ (heliocentric ecliptic latitude; Lallement et al. 2005). The components of the velocity in the ecliptic coordinates with the x -axis aligned towards the actual equinox $\vec{v}_F = -26.3 \text{ km/s} [\cos(254.7^\circ) \cos(5.2^\circ), \sin(254.7^\circ) \cos(5.2^\circ), \sin(5.2^\circ)]$ were transformed into a coordinate system with the xy -plane lying in the orbital plane of planet Neptune and the x -axis lying in the ecliptic plane. The orbital elements of the dust particle were determined in this reference coordinate system. The drag coefficients for Eq. (30) were calculated from Eq. (31). We assumed that the atoms are specularly reflected at the surface of the dust grain ($\delta_i = 1$). The planet was initially located on the x -axis. Initial semimajor axis of the dust particle is, in general, computed from the relation $a_{\beta \text{ in}} = a_{\beta \text{ nP}/n} + \Delta$, where $a_{\beta \text{ nP}/n}$ is defined by Eq. (7) (with β calculated from Eq. 3) and Δ is a shift from the exact resonant semimajor axis. As the initial conditions for a dust particle with $R = 2 \text{ }\mu\text{m}$, mass density $\varrho = 1 \text{ g/cm}^3$, and $\bar{Q}'_{pr} = 1$, we used $a_{\beta \text{ in}} = a_{\beta \text{ 2/1}} + 0.001 \text{ AU}$, $e_{\beta \text{ in}} = 0.5$, $\omega_{\beta \text{ in}} = 60^\circ$, $\Omega_{\beta \text{ in}} = 40^\circ$, and $i_{\beta \text{ in}} = 20^\circ$. The initial true anomaly of the dust particle was $f_{\beta \text{ in}} = 345^\circ$. From the numerical solution of Eq. (30) we obtained evolutions of the orbit averaged orbital elements and the orbit averaged time derivatives of orbital elements. Values of Tisserand parameter were determined in every time step from the oscular orbital elements. From these values evolution of the orbit averaged Tisserand parameter and the orbit averaged time derivative of Tisserand parameter was also obtained. Evolutions obtained from the numerical solution of Eq. (30) are depicted in Fig. 1 (black line) and Fig. 2 (grey line). We must note that the evolution duration 10^6 years requires the size of an interstellar gas cloud 26.9 pc in the direction of the interstellar gas velocity vector (constant velocity with magnitude 26.3 km/s is assumed) and such situation cannot always occur in the real galactic environment. To compare the numerical results with our analytical theory we calculated values of the secular time derivative of Tisserand parameter from Eq. (28) using the numerically calculated secular orbital elements. The obtained evolution is depicted in the bottom right panel of Fig. 2 using black line. The lines are practically overlapping each other. We also calculated values of the secular time derivative of eccentricity from equation (see Eq. 28)

$$\left\langle \frac{de_\beta}{dt} \right\rangle = \frac{1}{\partial C_T / \partial e_\beta} \left\langle \frac{dC_T}{dt} \right\rangle - \frac{\partial C_T / \partial i_\beta}{\partial C_T / \partial e_\beta} \left\langle \frac{di_\beta}{dt} \right\rangle, \quad (33)$$

where $\langle dC_T/dt \rangle$ was calculated from Eq. (28) and as $\langle di_\beta/dt \rangle$ we used the values obtained from numerical solution of Eq. (30) and depicted in the bottom left panel of Fig. 2. Obtained evolution is shown in the top right panel of Fig. 2

with black line. The analytical and numerical results are in excellent agreement.

4.3 Comparing analytical and numerical results in the planar case

In the planar case it is possible to compare directly the secular time derivative of eccentricity obtained from numerical solution with the same result obtained from analytical theory.

In the case when the dust particle captured in an MMR is only under the action of the PR effect and radial solar wind, secular time derivative of eccentricity is given by the first term in Eq. (29), namely

$$\left\langle \frac{de_\beta}{dt} \right\rangle = \beta \frac{\mu}{c} \left(1 + \frac{\eta}{\bar{Q}_{pr}} \right) \frac{(1 - e_\beta^2)^{1/2}}{a_\beta^2 e_\beta} \times \left[1 - \frac{2 + 3e_\beta^2}{2(1 - e_\beta^2)^{3/2}} \frac{n}{n_P} \right]. \quad (34)$$

Function $l(e) = (2 + 3e_\beta^2) / [2(1 - e_\beta^2)^{3/2}]$ is an increasing function of eccentricity. Function obtains values from $l(0) = 1$ to $\lim_{e \rightarrow 1} l(e) = \infty$. Therefore, the secular eccentricity is always a decreasing function in an interior resonance. In an exterior resonance such value of eccentricity $e_{\beta c}$ exists that

$$l(e_{\beta c}) = \frac{2 + 3e_{\beta c}^2}{2(1 - e_{\beta c}^2)^{3/2}} = \frac{n_P}{n} = \frac{p + q}{p}. \quad (35)$$

For $e_\beta < e_{\beta c}$ the secular eccentricity asymptotically increases to $e_{\beta c}$ and for $e_\beta > e_{\beta c}$ the secular eccentricity asymptotically decreases to $e_{\beta c}$. In the special case mean motion 1/1 resonance secular eccentricity asymptotically decreases to $e_{\beta c} = 0$ (Klačka et al. 2008; Pástor et al. 2009b). To visualise the secular evolution of eccentricity of the dust particle captured in an MMR only under the action of the PR effect and radial solar wind we numerically solved Eq. (30) without the third term. The planet was initially on the x -axis. We used dust particle with $R = 2 \mu\text{m}$, $\rho = 1 \text{ g/cm}^3$ and $\bar{Q}'_{pr} = 1$. The initial semimajor axis was $a_{\beta in} = a_{\beta 2/1} + 0.001 \text{ AU}$. The initial eccentricities and arguments of perihelion were 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9 and 260° , 275° , 285° , 295° , 295° , 290° , 280° , 265° , 250° , respectively. The initial true anomaly was $f_{\beta in} = 0$ for all particles. Results are depicted in Fig. 3. The limiting value $e_{\beta c}$ for mean motion 2/1 resonance obtained from Eq. (35) is $e_{\beta c} \approx 0.4812$. As can be easily seen from Fig. 3 the secular eccentricity really approaches this value.

An orbital evolution of the dust particle under the action of the PR effect, radial solar wind and IGF in the planar case is depicted in Figs. 4 and 5. We used the same parameters for various gas components of the IGF, initial position of the planet and properties of the dust grain as in Figs. 1 and 2. The planar case was attained by a rotation of the interstellar gas velocity vector into the orbital plane of Neptune around axis lying in the orbital plane of Neptune and perpendicular to the interstellar gas velocity vector. The ro-

tation angle was 3.7° . Initial orbital parameters of the dust grain were $a_{\beta in} = a_{\beta 2/1} + 0.001 \text{ AU}$, $e_{\beta in} = 0.3$, $\omega_{\beta in} = 0$ and $f_{\beta in} = 160^\circ$. As can be seen in Fig. 4 in this case the secular evolution of eccentricity is not a monotonic function of time. The secular time derivatives of eccentricity obtained from this numerical integration and from Eq. (29) are compared in Fig. 5. Again analytical theory and numerical solution are in excellent agreement.

4.4 Qualitative properties of orbital evolution in the planar case

In order to find qualitative properties of the dust particle orbital evolution in MMRs under the action of the PR effect, radial solar wind and IGF numerical solution Eq. (30) for many various initial conditions is needed. Calculation of the variable drag coefficients according to Eq. (31) is very time consuming therefore we used an approximation that the drag coefficients are constant. If the inequality $|\vec{v}| \ll v_F$ holds during orbit, then this approximation is usable (see Pástor 2012b). Many sets of initial conditions in Eq. (30) with the constant drag coefficients were numerically solved in order to study evolution of the dust particle in an exterior MMR. One such set of numerical solutions is depicted in Fig. 6. We used the same properties of the IGF, initial position of the planet and properties of the dust grain as in Figs. 4 and 5. Initial orbital parameters of the dust grain were $a_{\beta in} = a_{\beta 2/1} + 0.001 \text{ AU}$, $e_{\beta in} \in \{0.1, 0.2, \dots, 0.9\}$, $\omega_{\beta in} \in \{0, 5^\circ, 10^\circ, \dots, 355^\circ\}$ and $f_{\beta in} = 0$. Therefore, we obtained 648 individual evolutions. Not all 648 evolutions are depicted in Fig. 6. An evolution is depicted in Fig. 6 only if the dust particle was captured and remained in the resonance longer than 10^5 years. Each evolution depicted in Fig. 6 is represented by a marker in Fig. 7. Position of each marker corresponds to the initial eccentricity and initial argument of perihelion. Colour of each marker corresponds to the capture time for given evolution. White colour would theoretically correspond to zero capture time and the darkest shade of a given colour corresponds to the capture time 2×10^6 years. A place in Fig. 7 without marker corresponds to the initial eccentricity and initial argument of perihelion for which the dust particle was not captured in the resonance or remained in the resonance shorter than 10^5 years. 393 evolutions is depicted in Fig. 6. From many such sets of numerical solutions as set depicted in Fig. 6 we found that two main groups of orbital evolution in an exterior MMR under the action of the PR effect, radial solar wind and IGF exist. As was already mentioned all evolutions depicted in Fig. 6 have the initial true anomaly equal to zero. For sets with different initial value of the true anomaly are these two main groups preserved. These two groups have also longest capture times.

Orbital evolutions in the first group are characterised by approach of the eccentricity and argument of perihelion to some “constant” values. We found that these values are not exactly constant but in comparison with the evolution be-

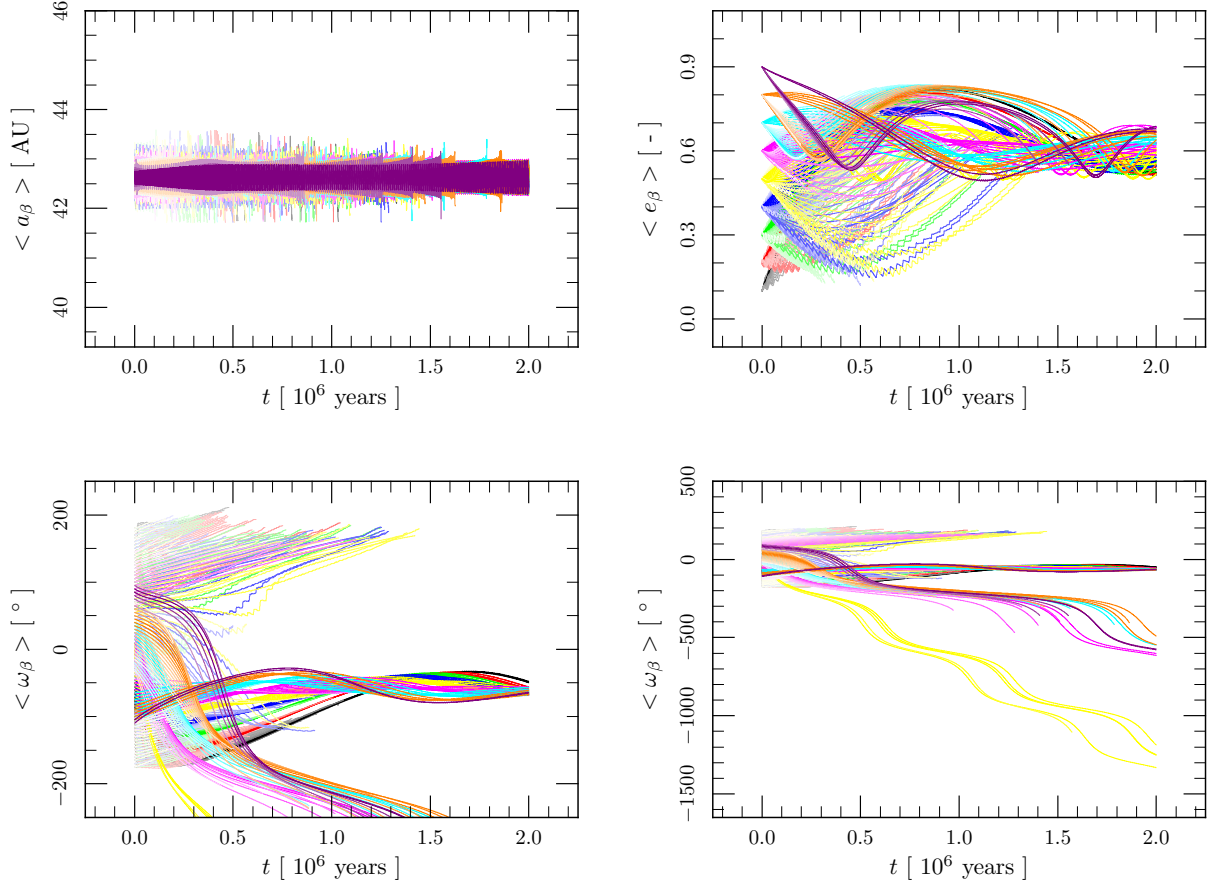


Figure 6. Evolutions of the orbit averaged Keplerian orbital elements of a dust particle with $R = 2 \mu\text{m}$, $\rho = 1 \text{ g/cm}^3$, and $\bar{Q}'_{pr} = 1$ captured in an exterior mean motion orbital 2/1 resonance with Neptune under the action of the PR effect, radial solar wind and IGF in the planar case. The initial values of orbital parameters were $a_{\beta \text{ in}} = a_{\beta \text{ 2/1}} + 0.001 \text{ AU}$, $e_{\beta \text{ in}} \in \{0.1, 0.2, \dots, 0.9\}$, $\omega_{\beta \text{ in}} \in \{0, 5^\circ, 10^\circ, \dots, 355^\circ\}$ and $f_{\beta \text{ in}} = 0$. Depicted are only the evolutions with the capture time longer than 10^5 years (see Fig. 7).

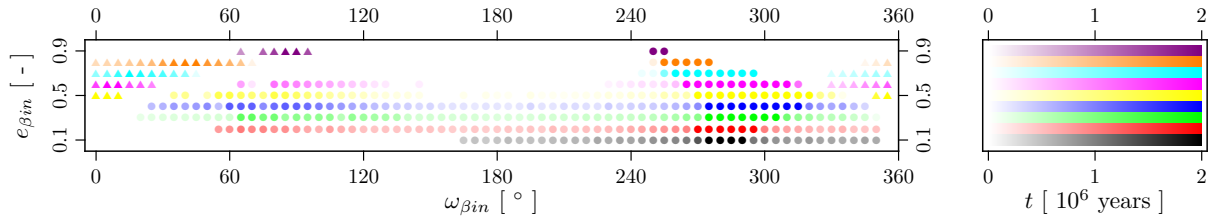


Figure 7. The initial eccentricities and arguments of perihelion of the evolutions depicted in Fig. 6. Colour of each marker, on a position given by the initial eccentricity and argument of perihelion, corresponds the capture time of obtained evolution. The capture times are from 0 to 2×10^6 years and the colours are from white to the darkest shade of a given colour, respectively. If the capture time is shorter than 10^5 years, then the corresponding position is empty. The initial conditions of evolutions belonging to the second group (evolutions with the fast monotonic shift of perihelion and oscillations of eccentricity) are marked with triangles (see text).

fore this “stabilisation” are relatively slowly changing. The stabilised direction of the position vector of the orbit’s perihelion is almost parallel with direction of the IGF velocity vector in Fig. 6. The parallel case corresponds to the argu-

ment of perihelion $\omega_{\beta} \approx -57.16^\circ$ in Fig. 6. Many evolutions approach to this value of ω_{β} as can be seen in the bottom left panel of Fig. 6. $I_{\beta} = 0$ in the parallel case (see Eq. 27). If we use in Eqs. (16)-(20) condition $\sigma_{\beta} = 0$, we obtain the sec-

ular time derivatives of orbital elements caused by the PR effect, radial solar wind and IGF described by a constant acceleration. If the dust particle is only under the action of the solar gravity (which can be reduced using the radial Keplerian term of the PR effect) and the constant acceleration caused by the IGF, then the secular orbital motion can be completely solved analytically (see Pástor 2012a; Belyaev & Rafikov 2010). Perhaps, the following idea can appear: for constant acceleration caused by the IGF ($\sigma_\beta = 0$) the stabilisation of eccentricity would correspond to trivial solution of Eq. (29) $I_\beta = 0$ and $e_\beta = e_{\beta c}$ (Eq. 35). Therefore, we solved Eq. (30) with acceleration caused by the IGF described by a constant vector (dependence on particle velocity in the acceleration was not included) for a set of initial conditions identical to the set used in Fig. 6. The stabilisation occurred also in this case. However, the argument of perihelion into which perihelia approached was not equal to -57.16° but -63° . Another important difference was in the evolution eccentricity. The eccentricities approached to $e_\beta \approx 0.61$ and not to $e_{\beta c} \approx 0.4812$. Therefore, at least in this simplified case, the stabilisation does not correspond to trivial solution of Eq. (29) $I_\beta = 0$ and $e_\beta = e_{\beta c}$.

Orbital evolutions in the second group are characterised by a fast monotonic shift of perihelion and oscillations of eccentricity. Typical examples of this group are depicted in the bottom right panel of Fig. 6 as rapidly decreasing curves. The initial conditions which lead to this behaviour are marked with triangles in Fig. 7.

5 CONCLUSION

We investigated the orbital evolution of a dust particle captured in an MMR with a planet in a circular orbit under the action of the PR effect, radial stellar wind and IGF. The secular time derivative of Tisserand parameter is analytically derived for arbitrary orbit orientation using previously derived secular time derivatives of Keplerian orbital elements caused by the PR effect, radial stellar wind and IGF. From the secular time derivative of Tisserand parameter a relation between the secular time derivatives of eccentricity and inclination can be obtained. In the planar case we derived directly the secular time derivative of eccentricity.

We numerically solved equation of motion of the dust particle in order to compare the analytically derived results with the numerically obtained results in the three-dimensional case and also in the planar case. In both cases analytical and numerical results are in excellent agreement. This implies that the theory developed in Liou & Zook (1997) for the PR effect and radial solar wind can be generalised for arbitrary non-gravitational effects with known secular time derivatives.

If the dust particle is captured in an exterior MMR in the planar CR3BP with the PR effect and radial stellar wind, then the secular eccentricity is a monotonic function of time. If we take into account also an IGF, complicated behaviour with oscillations shows. This is most likely caused

by dependence of the secular time derivative of eccentricity on the argument of pericentre due to directional character of the acceleration caused by the IGF. However, qualitative properties of the secular evolution of eccentricity and argument of pericentre can be determined. We found that two main groups exist. In the first group the eccentricity and argument of pericentre approach to some values. In the second group the eccentricity oscillates and argument of pericentre rapidly shifts.

REFERENCES

- Alouani-Bibi F., Opher M., Alexashov D., Izmodenov V., Toth G., 2011. Kinetic versus multi-fluid approach for interstellar neutrals in the heliosphere: Exploration of the interstellar magnetic field effects. *Astrophys. J.* **734**, 45.
- Baines M. J., Williams I. P., Asebiomo A. S., 1965. Resistance to the motion of a small sphere moving through a gas. *Mon. Not. R. Astron. Soc.* **130**, 63-74.
- Belyaev M., Rafikov R., 2010. The dynamics of dust grains in the outer Solar System. *Astrophys. J.* **723**, 1718-1735.
- Brouwer D., Clemence G. M., 1961. *Methods of Celestial Mechanics* Academic Press, New York.
- Brownlee D. E., 1994. The ring around us. *Nature* **369**, 706.
- Bruno R., Carbone V., Sorriso-Valvo L., Bavassano B., 2003. Radial evolution of solar wind intermittency in the inner heliosphere. *J. Geophys. Res.* **108**, A3, 1130.
- Deller A. T., Maddison S. T., 2005. Numerical modeling of dusty debris disks. *Astrophys. J.* **625**, 398-413.
- Dermott S. F., Jayaraman S., Xu Y. L., Gustafson B. A. S., Liou J. C., 1994. A circumsolar ring of asteroidal dust in resonant lock with the Earth. *Nature* **369**, 719-723.
- Ertel S., Wolf S., Rodmann J., 2012. Observing planet-disk interaction in debris disks. *Astron. Astrophys.* **544**, A61.
- Frisch P. C., Bzowski M., Grün E., Izmodenov V., Krüger H., Linsky J. L., McComas D. J., Möbius E., Redfield S., Schwadron N., Shelton R., Slavin J. D., Wood B. E., 2009. The galactic environment of the Sun: Interstellar material inside and outside of the heliosphere. *Space Sci. Rev.* **146**, 235-273.
- Gustafson B. A. S., 1994. Physics of Zodiacal Dust. *Annu. Rev. Earth Planet. Sci.* **22**, 553-595.
- Holmes E. K., Dermott S. F., Gustafson B. A. S., Grogan K., 2003. Resonant structure in the Kuiper disk: An asymmetric Plutino disk. *Astrophys. J.* **597**, 1211-1236.
- Jackson A. A., Zook H. A., 1989. A Solar System dust ring with the Earth as its shepherd. *Nature* **337**, 629-631.
- Klačka J., 1992. Poynting-Robertson effect. I. Equation of motion. *Earth, Moon, and Planets* **59**, 41-59.
- Klačka J., 2004. Electromagnetic radiation and motion of a particle. *Celest. Mech. Dyn. Astron.* **89**, 1-61.
- Klačka J., Kocifaj M., Pástor P., 2005a. Motion of dust near exterior resonances with planets. *J. Phys.: Conf. Ser.* **6**, 126-131.
- Klačka J., Kocifaj M., Pástor P., 2005b. Effect of radiation

- on nonspherical particles in resonances with large planets. In: Moreno F., López-Moreno J. J., Muñoz O., Molina A. (eds.), *8th Conference on Electromagnetic and Light Scattering by Nonspherical Particles: Theory, Measurements and Applications* Instituto de Astrofísica de Granada, pp. 156-159.
- Klačka J., Kómar L., Pástor P., Petržala J., 2008. The non-radial component of the solar wind and motion of dust near mean motion resonances with planets. *Astron. Astrophys.* **489**, 787-793.
- Klačka J., Kómar L., Pástor P., Petržala J., 2009. Solar wind and motion of interplanetary dust grains. In: Johannson H. E. (ed.), *Handbook on Solar Wind: Effects, Dynamics and Interactions* NOVA Science Publishers, New York, pp. 227-273.
- Klačka J., Petržala J., Pástor P., Kómar L., 2012a. Solar wind and motion of dust grains. *Mon. Not. R. Astron. Soc.* **421**, 943-959.
- Klačka J., Petržala J., Pástor P., Kómar L., 2012b. The Poynting-Robertson effect: A critical perspective. *Icarus*, in press, doi:10.1016/j.icarus.2012.06.044
- Krivov A. V., Queck M., Löhne T., Sremčević M., 2007. On the nature of clumps in debris disks. *Astron. Astrophys.* **462**, 199-210.
- Kuchner M. J., Holman M. J., 2003. The geometry of resonant signatures in debris disks with planets. *Astrophys. J.* **588**, 1110-1120.
- Lallement R., Quémerais E., Bertaux J.L., Ferron S., Koutroumpa D., Pellinen R., 2005. Deflection of the interstellar neutral hydrogen flow across the heliospheric interface. *Science* **307**, 1447-1449.
- Liou J.-Ch., Zook H. A., 1995. An asteroidal dust ring of micron-sized particles trapped in the 1:1 mean motion resonance with Jupiter. *Icarus* **113**, 403-414.
- Liou J.-Ch., Zook H. A., 1997. Evolution of interplanetary dust particles in mean motion resonances with planets. *Icarus* **128**, 354-367.
- Liou J.-Ch., Zook H. A., 1999. Signatures of the giant planets imprinted on the Edgeworth-Kuiper belt dust disk. *Astron. J.* **118**, 580-590.
- Liou J.-Ch., Zook H. A., Jackson A. A., 1995. Radiation pressure, Poynting-Robertson drag, and solar wind drag in the restricted three body problem. *Icarus* **116**, 186-201.
- Marzari F., 2012. Interstellar medium perturbations on transport-dominated debris discs in binary star systems. *Mon. Not. R. Astron. Soc.* **421**, 3431-3442.
- Marzari F., Thébault P., 2011. On how optical depth tunes the effects of the interstellar medium on debris discs. *Mon. Not. R. Astron. Soc.* **416**, 1890-1899.
- Marzari F., Vanzani V., 1994. Dynamical evolution of interplanetary dust particles. *Astron. Astrophys.* **283**, 275-286.
- Moro-Martín A., Malhotra R., 2002. A study of the dynamics of dust from the Kuiper belt: spatial distribution and spectral energy distribution. *Astron. J.* **124**, 2305-2321.
- Mustill A. J., Wyatt M. C., 2011. A general model of resonance capture in planetary systems: first- and second-order resonances. *Mon. Not. R. Astron. Soc.* **413**, 554-572.
- Pástor P., 2012a. Influence of fast interstellar gas flow on the dynamics of dust grains. *Celest. Mech. Dyn. Astron.* **112**, 23-45.
- Pástor P., 2012b. Orbital evolution under the action of fast interstellar gas flow with a non-constant drag coefficient. *Mon. Not. R. Astron. Soc.* **426**, 1050-1060.
- Pástor P., Klačka J., Kómar L., 2009a. Motion of dust in mean motion resonances with planets. *Celest. Mech. Dyn. Astron.* **103**, 343-364.
- Pástor P., Klačka J., Kómar L., 2011. Orbital evolution under the action of fast interstellar gas flow. *Mon. Not. R. Astron. Soc.* **415**, 2637-2651.
- Pástor P., Klačka J., Petržala J., Kómar L., 2009b. Eccentricity evolution in mean motion resonance and non-radial solar wind. *Astron. Astrophys.* **501**, 367-374.
- Poynting J. M., 1904. Radiation in the Solar System: its effect on temperature and its pressure on small bodies. *Philos. Trans. R. Soc. Lond. Ser. A* **202**, 525-552.
- Reach W. T., 2010. Structure of the Earth's circumsolar dust ring. *Icarus* **209**, 848-850.
- Reach W. T., Franz B. A., Welland J. L., Hauser M. G., Kelsall T. N., Wright E. L., Rawley G., Stemwedel S. W., Splesman W. J., 1995. Observational confirmation of a circumsolar dust ring by the COBE satellite. *Nature* **374**, 521-523.
- Robertson H. P., 1937. Dynamical effects of radiation in the Solar System. *Mon. Not. R. Astron. Soc.* **97**, 423-438.
- Scherer K., 2000. Drag forces on interplanetary dust grains induced by the interstellar neutral gas. *J. Geophys. Res.* **105**, A5, 10329.
- Šidlichovský M., Nesvorný D., 1994. Temporary capture of grains in exterior resonances with Earth: Planar circular restricted three-body problem with Poynting-Robertson drag. *Astron. Astrophys.* **289**, 972-982.
- Stark C. C., Kuchner M. J., 2008. The detectability of exo-Earths and super-Earths via resonant signatures in exozodiacal clouds. *Astrophys. J.* **686**, 637-648.
- Tisserand F. F., 1896. *Traité de Mécanique Céleste IV* Gauthier-Villars, Paris.
- Weidenschilling S. J., Jackson A. A., 1993. Orbital resonances and Poynting-Robertson drag. *Icarus* **104**, 244-254.
- Whipple F. L., 1955. A comet model III. The zodiacal light. *Astrophys. J.* **121**, 750-770.
- Wyatt S. P., Whipple F. L., 1950. The Poynting-Robertson effect on meteor orbits. *Astrophys. J.* **111**, 134-141.